Also, Paolucci states that "no constraint is placed on how the metrics are to be evaluated." In so far as the metrics x_ξ, x_η, x_ζ, y_ξ, y_η, y_ζ, z_ξ, z_η, z_ζ are concerned, this statement is true for the three-dimensional case. However, the inverse metrics ξ_x, ξ_y, ξ_z, η_x, η_y, η_z, ζ_x, ζ_y, ζ_z are most certainly constrained by the consistency requirement, even though the transformed governing equations may be formulated without the explicit appearance of these "inverse metrics."

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Comment on "Minimum-Weight Design of an Orthotropic Shear Panel with Fixed Flutter Speed"

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IN Ref. 1, Beiner and Librescu have presented an analysis of weight minimization for rectangular flat panels in a high supersonic flowfield subject to a panel flutter speed constraint based on aerodynamic piston theory. To simplify the problem, they have used a structural model that considers transverse shear deformation only and neglects the bending stiffness of the plate. This has the effect of reducing the linear partial differential equation for the panel flutter problem from fourth to second order. The determination of the panel flutter speed in the analysis is based on the application of Galerkin's method.

Unfortunately, as is already well-known for the problem of membrane flutter, also governed by a second-order differential equation, such reduction in the order of the differential equation may lead to difficulties in the convergence of Galerkin's method. In fact, for the two-dimensional membrane panel flutter problem with supersonic aerodynamic forces respresented by piston theory, Galerkin's method leads to finite flutter speeds for any finite number of Galerkin weighting functions, whereas, as shown by a number of authors,[2,3,4] the exact solution of the differential equation (which is, of course, itself approximate) shows the panel to be stable for all supersonic flight velocities high enough for piston theory to apply.

An examination of Beiner and Librescu's Eq. (1) shows that, like the equation for membrane flutter, it must lead to exact solutions for frequency which are real under all supersonic flight conditions to which piston theory applies. Therefore, panel flutter speeds derived by Galerkin's method are spurious. The general demonstration of this property of the differential equation (i.e., that it can have no other than real eigenvalues) follows from the proof that for ordinary linear differential equations with variable real coefficients any second-order equation may be put in self-adjoint form by multiplying it by the non-vanishing function

$$I(x) = \frac{1}{a_0} \exp \int_{x_0}^{x} \frac{a_1(\rho)}{a_0(\rho)} \, d\rho$$

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where x is the independent variable, $a_0(x)$ is the coefficient of the second derivative term, and $a_1(x)$ is the coefficient of the first derivative term. In addition, the boundary conditions must satisfy the conditions for self-adjointness of the differential system.[5] Since in the present case of a partial differential equation the term in the lateral dimension is already in the self-adjoint form, the reduction in the streamwise direction, which has both a first and a second derivative term, may be carried out exactly as in the case of an ordinary differential equation, rendering the entire equation self-adjoint.

The self-adjoint properties of the partial differential equation for panel flutter when only transverse shear deformations are accounted for can easily be seen for the special case of constant panel thickness. For this case, Eq. (1) of Ref. 1 becomes

$$\bar{G}_{13} \frac{\partial^2 w}{\partial y^2} + \bar{G}_{23} \frac{\partial^2 w}{\partial x^2} - \lambda \frac{\partial w}{\partial x} + \omega^2 \mu w = 0 \tag{1}$$

where, as in the original paper, $\bar{G}_{13}$ and $\bar{G}_{23}$ are, respectively, the elastic moduli in transverse shear in the lateral and longitudinal directions nondimensionalized with respect to a convenient shear modulus, G_{ref}, and direction 3 corresponds to the direction transverse to the plate. $\lambda = \kappa p_\infty M_\infty / G_{\text{ref}} h$, where κ is the polytropic gas coefficient, and p_∞ and M_∞ are the pressure and Mach number, respectively, in the undisturbed flow. $\mu = \rho / G_{\text{ref}}$ where ρ is the mass density of the plate. The motion in time is taken to be given by $\bar{w} = we^{i\omega t}$, where ω is the circular frequency.

If b is the transverse dimension of the plate, Eq. (1) has a solution of the form

$$w = \frac{\sin m\pi y}{b} F(x)$$

where m is any integer, so that

$$\bar{G}_{23} \frac{d^2 F}{dx^2} - \lambda \frac{dF}{dx} + \left(\omega^2 \mu - \frac{m^2 \pi^2}{b^2} \bar{G}_{13} \right) F = 0 \tag{2}$$

This may be further reduced by substituting

$$F(x) = f(x) \exp \frac{\lambda}{2\bar{G}_{23}} x$$

to give

$$\bar{G}_{23} \frac{d^2 f}{dx^2} + \left(\omega^2 \mu - \frac{m^2 \pi^2}{b^2} \bar{G}_{13} - \frac{\lambda^2}{4\bar{G}_{23}} \right) f = 0 \tag{3}$$

Finally the solution of this obviously self-adjoint equation is given by

$$f = \sin(n\pi x / a)$$

where a is the dimension of the panel in the x direction and n may be any integer. Thus

$$\omega^2 = \frac{1}{\mu} \left[\frac{n^2 \pi^2}{a^2} \bar{G}_{23} + \frac{m^2 \pi^2}{b^2} \bar{G}_{13} + \frac{\lambda^2}{4\bar{G}_{23}} \right] \tag{4}$$

The frequencies of the panel are seen to be all real and all increase monotonically with the aeroelastic Mach number parameter, λ. No panel flutter is possible for any value of λ.

Bolotin[4] has discussed the convergence (or lack thereof) of the Galerkin method for the membrane of infinite aspect ratio in terms of the properties of the infinite determinant generated by Galerkin's method with an infinite number of weighting functions. He shows that the infinite determinant does not converge for the membrane but that with finite plate bending stiffness (i.e., a fourth order differential equation) the determinant converges although ever more slowly as the membrane stiffness contribution to frequency becomes high compared to the bending stiffness contribution. Similar arguments apply to the transverse shear model as bending stiffness is introduced into the model.

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Reply by Authors to A. H. Flax

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BY developing an exact solution to Eq. (1) of Ref. 1 for the special case of uniform panel thickness, Dr. Flax points out that pure transverse shear panels (PTSP) cannot undergo flutter in the range of supersonic velocities for which piston theory applies. This is a paradoxical result, since it appears by considering, for example, the flutter of a sandwich-type panel consisting of two thin faces (modeled in accordance with Love-Kirchhoff theory), separated by a core modeled as a PTSP. As is well known, such a panel will flutter if the dynamic pressure becomes large enough. As the faces become thinner, the panel will flutter at lower dynamic pressures, i.e., become less stable. On the other hand, when in the limit the thickness of the faces vanishes (resulting in a structure with no bending stiffness), the flutter speed of the remaining PTSP becomes infinite according to Flax's exact solution. A paradox is thus reached in which a PTSP in high supersonic flow should be always stable, while more rigid structures (in bending) can flutter. This result belongs to the same class of aeroelastic paradoxes as the well-known membrane flutter paradox (see Refs. 2-4 of the Comment and our Refs. 2-4).

However, by using a singular perturbation method, a solution of the membrane flutter paradox has been developed in Ref. 5. This solution (which will be also useful for the problem at hand) considers the case of two-dimensional flat thin panels subjected to chordwise in-plane tensile stresses σ_x and exposed over its upper face to a supersonic flowfield. It is assumed that $D \equiv E'h^3$ is the panel bending stiffness, where $E' \equiv E/12(1-\nu^2)$ denotes the reduced Young's modulus. Linear piston aerodynamics is employed. An "interior solution" is first constructed by removing the exponential factor $\exp(\alpha x)$ appearing in the exact membrane flutter solution (a similar factor appears in the exact PTSP solution of the Comment); subsequently, the bending stresses near the edges are accounted for by a "boundary-layer" approach, and by letting $D \to 0$, the following flutter criterion is obtained

$$\frac{\rho_\infty U_\infty^2}{M_\infty \sigma_x}\left(\frac{E'}{\sigma_x}\right)^{1/2} = \left(\frac{2}{3}\right)^{3/2} \tag{1}$$

The above flutter criterion for "zero-thickness plates"[5] was obtained as a zero-order solution in the perturbation process. In Ref. 6 the same result was obtained from Erickson's three-dimensional panel flutter solution by a limiting process $D \to 0$. Note from Eq. (1) that the geometrical and mechanical characteristics in the spanwise direction are not intervening. Having in view the similarity between the aeroelastic equilibrium equations of flat panels with in-plane tensile stresses and sandwich-type panels with PTSP core, a flutter criterion for uniform PTSP can be obtained from Eq. (1) by replacing σ_x by G_{13}, thus becoming

$$(\Lambda_0)_* = \left(\frac{2}{3}\frac{G_{13}}{E'}\right)^{2/3} \tag{2}$$

where $\Lambda_0 = \kappa p_\infty M_\infty / E'$ defines a velocity parameter and E' denotes the reduced Young's modulus of the faces whose thickness becomes zero in the limiting process. Therefore, finite flutter speeds are predicted by viewing the uniform PTSP as the core of a symmetrical sandwich structure with the thickness of the faces tending to zero.

In the numerical example presented in Ref. 1, the critical flutter speed $(\Lambda_0)_*$ of the uniform PTSP—which intervenes as a fixed parameter in the optimal solution—was obtained by a Galerkin technique, where the representation of w corresponds to the interior solution of Ref. 5. It is of course advisable to use criterion (2)—believed by its authors[5] to be a crude but conservative one—for calculating the flutter speed of the uniform PTSP. It is hoped that the present discussion prompted by the Comment will stimulate the development of even more accurate solutions to the membrane and PTSP flutter problems by using higher order approximations in the perturbation method. This will also constitute better input data for the aeroelastic optimization problem considered in Ref. 1.

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The Nondimensional Coefficient of Thermal Conductivity

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THE purpose of this Note is to draw attention to the misleading notation of the coefficient of thermal conductivity divided by Prandtl number, κPr^{-1}, in many

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